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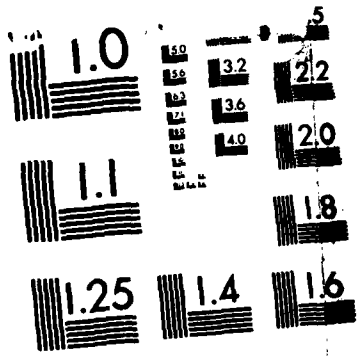

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The work in progress has included the result that, for a certain class of partial differential equations, nonlinearization will often not aid in stabilization. Partial results have been obtained on the following two types of questions: (1) What sort of estimates can one obtain for solutions of nonlinear parabolic problems backward in time at a given time in terms of the solution at alter times? Also: (2) If a solution of a certain parabolic boundary value problem is known to exist for positive time, what are the possible behaviors as the time tends to zero? Other topics addressed include coefficient determination in hyperbolic equations, quenching for wave equations, and positive solutions of a nonlinear elliptic equation in all of space. Two presentations will be given this summer on results to date.

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AFOSR-TR- 86-0047

Interim

Research Progress and Forecast Report

AFOSR -84-0252

April 15, 1985

Howard A. Levine
Principle Investigator

I. Completed Work (Papers in Preparation)

II. Work in Progress

A. Stabilization by nonlinearization

B. Nonlinear parabolic problems backward in time

C. Coefficient determination problems in hyperbole equations

D. Quenching for wave equations

E. Burger's equation

F. Positive solutions of nonlinear elliptic equations

III. Meetings and other Travel

IV. Personnel

V. Visitors to I.S.U.

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MATTHEW J. KEMPER
Chief, Technical Information Division

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I. Papers in Preparation (Work Completed)

1. (With H. F. Weinberger) An inequality between the eigenvalues of the Dirichlet and Neumann problems for the Laplacian.

If $\{\lambda_k\}_{k=1}^{\infty}$, $\{\mu_k\}_{k=1}^{\infty}$ denote the increasing sequences of eigenvalues of the Dirichlet and Neuman problems for a bounded, convex domain $D \subset R^N$ with C^2 boundary, then $\mu_{k+N} < \lambda_k$. (Payne proved this in 1955 for $N = 2$). If ∂D is not necessarily C^2, then $\mu_{k+N} \leq \lambda_k$. It is shown that the inequality may hold for some nonconvex domains and fail for others. If the domain is not convex but the mean curvature is positive on ∂D, then $\mu_{k+1} < \lambda_k$. Other examples are given.

(P. Aviles has applied these results to obtain a partial solution to Pompieu's problem (Schiffer's conjecture).)

2. (With R. A. Smith) A potential well theory for the wave eqution with a nonlinear boundary condition.

3. (With R. A. Smith) A potential well theory for the heat equation with a nonlinear boundary condition.

In this pair of papers, problems of the following type are considered: Let $D \subset R^N$, D bounded, $\partial D = \Sigma \cup \sigma$, $\sigma \cap \Sigma = \phi$. We seek a solution on $D \times [0,\infty)$ of

$$u_{tt} = \Delta u \quad \text{in} \quad D \times [0,\infty)$$

$$u = 0 \quad \text{on} \quad \Sigma \times [0,\infty)$$

$$\frac{\partial u}{\partial n} = u^p \quad \text{on} \quad \sigma \times [0,\infty)$$

with prescribed initial data, $u[x,0)$, $u_t(x,0)$. The potential energy

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$$J(u) = \frac{1}{2} \int_D |\nabla u|^2 - \frac{1}{p+1} \int_\sigma \varphi^{p+1} \, ds$$

is shown to be continuous in Dirichlet norm for $2 < p+1 < 2N/(N-2)$. A potential well is constructed (using trace inequalities) and the depth d is given by

$$d = \{J(\varphi) \mid \int_D |\nabla \varphi|^2 = \int_\sigma \varphi^{p+1} ds, \quad \varphi \in H^1_\sigma(D)\}$$

where $H^1_\sigma(D)$ is the completion in Dirichlet norm of $C^1(\bar{D})$ functions vanishing in Σ. We define

$$W^i = \{\varphi \in H^1_\sigma(D) \mid J(\varphi) < d, \quad \int_D |\nabla \varphi|^2 > \int_\sigma \varphi^{p+1} \, ds\}$$

$$W^e = \{\varphi \in H^1_\sigma(D) \mid J(\varphi) < d, \quad \int_D |\nabla \varphi|^2 < \int_\sigma \varphi^{p+1} \, ds\} .$$

The principle result is: If $u(x,0) \in W^i$, $E(0) = J(u(\cdot,0)) + \frac{1}{2} \int_D u_t^2(x,0)dx < d$ then we have a global solution. If $E(0) < d$ and $u(x,0) \in W^e$, then the solution cannot exist for all t.

The result is analogous to that of Payne and Sattinger for the nonlinear wave equation $u_{tt} = \Delta u + u^p$. In order to demonstrate global existence, in addition to the implementation of trace inequalities, the approximating solutions have to be taken to be of the form

$$u_{nm}(x,t) = \sum_{i=1}^{n} p_i(t) \varphi_i(t) + \sum_{j=1}^{m} q_j(t) \psi_j(x)$$

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where the φ_i are the eigenfunctions for the problem

$$\Delta\varphi + \lambda\varphi = 0 \qquad \text{in } D$$

$$\varphi = 0 \qquad \text{on } \Sigma$$

$$\frac{\partial\varphi}{\partial n} = 0 \qquad \text{on } \sigma$$

whilst the ψ_i's are the eigenfunctions for the Steklov type problem.

$$\Delta\psi = 0 \qquad \text{in } D$$

$$\psi = 0 \qquad \text{in } \Sigma$$

$$\frac{\partial\psi}{\partial n} = \lambda\psi \qquad \text{on } \sigma .$$

In the original analysis of Sattinger, only the first sum was needed. (Several errors in Payne and Sattinger [Is. J. Math, 22(1975), 273-303] are corrected.)

II. Work in Progress

A. Stabilization by Nonlinearization

The problems under consideration here are illustrated following where $D \subset R^2$ is a domain with a piecewise smooth boundary, ∂D:

$$u_{tt} = u_{xx} - u_{yy} - u^3 \qquad \text{in } D \times [0,\infty)$$

$$u(x,y,t) = 0 \qquad \text{for } (x,y) \in \partial D$$

$$u(x,y,0),\ u_t(x,y,0) \qquad \text{prescribed.}$$

The linear version of the above problem is ill posed. The hope was that by putting a nonlinear term of the right sign in the linear equation, one could

stabilize this problem. However, for the one dimensional problem

$$u_{tt} = - u_{xx} - u^3 \qquad 0 < x < \pi, \quad t > 0.$$

$$u(0,t) = u(\pi,t) = 0$$

$$u(x,0),\ u_t(x,0) \qquad \text{prescribed}$$

we have shown, first numerically, and then theoretically that this procedure is doomed to failure. Indeed, all stationary solutions are unstable. (The linearized problem about every stationary point is unstable.)

The two dimensional problem is still of interest however, because while every stationary solution is still unstable, one can show that every such solution is a saddle point. Moreover, as Brezis has observed, it is not clear if there are stationary solutions other than $u \equiv 0$.

To see more clearly what is going on, if one considers $D = [0,\pi] \times [0,\pi]$ and looks at the problem with boundary data

$$u_y(x,0,t) = u_y(x,\pi,t) = 0$$

$$u(0,y,t) = u(\pi,y,t) = 0$$

instead of the Dirichlet problem, then global (in time) solutions (depending only on (x,t)) do exist. Thus, with more stringent boundary conditions, we might expect this to be the case for the original problem. These points will be pursued further.

B. <u>The behavior of nonlinear parabolic problems backward in time</u>. (Paul Sacks).

Partial results have been obtained on the following two types of questions. (i) What sort of estimates can one obtain for solutions at a given time in terms of the solution at later times? Aside from their own intrinsic interest, such estimates are relevant to some types of control problems. (ii)

If a solution of a certain parabolic boundary value problem is known to exist for positive time, what are the possible behaviors as the time tends to zero? Can there be solutions which do not arise as solutions of initial and boundary value problems in the ordinary way?

I hope to pursue both of these questions as well as related ones which may suggest themselves.

C. Coefficient determination in hyperbolic equations. (Paul Sacks)

Such problems arise in geophysics etc. where material properties of an object are to be determined from indirect measurements. Recent theoretical investigations have suggested solution methods. which might be practically implemented. Furthermore, it is of interest to compare the predictions made by the theory with particular calculations which could be done numerically. While working with the Cornell group on O.N.R. sponsored research on inverse problems, I have become involved in several projects of this type, and hope to spend some time continuing these investigations.

D. Quenching for wave equations.

Smith has shown numerically that some solutions of the initial-boundary value problem for $u_{tt} = \Delta u + \varepsilon/(1-u)$ in the unit N-ball do not quench (reach one in a finite or infinite time) when ε is small. However, we were not able to use potential well arguments to prove this result. In general, one does not have an embedding form L^{∞} into H_0^1 except in one dimension. Nevertheless, it seems that when

$$\frac{1}{(u_{max}(t))^2} \int_{Ball} |\nabla u|^2 dx$$

is initially small the solution quenches, while if it is initially large the

solution exists for all time. We suspect that for solutions that do not quench, there is a constant (depending on the data) which bounds the ratio from below. We are pursuing this idea further.

E. The Burger's Equation $u_t + uu_x = \alpha u_{xx} + \beta u^3$.

We have shown (via energy arguments) that the solution $u = 0$ of the initial boundary value problem is stable. However, this is only a local result. T. S. Chen will write a routine which will allow us to examine what happens for "arbitrary" initial data. She has begun to program the problem using a fully implicit finite difference marching scheme.

(The stability result was obtained by Brian Straughan and me. He has observed that the p.d.e. is analogous to to one which arises in the study of penetrative convection of a heavy fluid which lies above a lighter one.)

F. Positive solutions of a nonlinear elliptic equation in all of space.

Our masters student, Thomas K. Evers, is trying to solve the following boundary value problem for a positive solution:

$$r^{-(n-1)} \frac{d}{dr} \left(\frac{y' r^{n-1}}{(1+(y')^2)^{1/2}} \right) + \lambda y^q - y = 0; \quad 0 < r < \infty$$

$$y'(0) = u(\infty) = 0 .$$

Here $q > 1$ and

$$\lambda > \left(\frac{1}{2} \frac{q-1}{q+1}\right)^{\frac{1}{2}(q-1)},$$

a range of parameters for which Serrin and Ni (Minnesota) are unable to prove the nonexistence of such solutions.

Preliminary results indicate that such solutions exist. However, the equation has very stiff solutions and further experimentation is necessary.

Evers will work further on this problem during the summer. He expects to graduate in August.

III. Meeting and other Travel Plans

1. R. A. Smith will speak on the results of his thesis (papers 2,3) at the international Conference on Theory and Applications of Differential Equations, May 20-23, Edinburgh, Texas.

2. H. A. Levine will visit Cornell University, June 22-29 to discuss grant related problems with L. E. Payne and other numerical analysts there.

3. Tsu Fen Chen will attend the SIAM 1985 Spring meeting in Pittsburgh, June 24-26. She will discuss least square approximations to compressible flow problems.

IV. Personnel.

1. Professor Howard A. Levine, P.I. - (one academic month, June, July 1985)

2. Assistant Professor Paul E. Sacks (co P.I.) - (May 15 - Aug. 15, 1985)

3. Assistant Professor Tsu Fen Chen - (June, July 1985)

4. Richard A. Smith, R.A. (Ph.D. student) - (October 1, 1984 - Oct. 1, 1985)

5. Thomas K. Evers, R.A. (M.S. student) - (June, July 1985)

V. Visitors to I.S.U. supported in part by AFOSR-84-0252

1. Ralph Showalter, Mathmatics Department, University of Texas, Austin. (10/2/84). Colloquium lecture; consulted with HAL on various ill posed problems.

2. Brian Straughan, Mathematics Department, University of Wyoming, (visitor) (10/23/85). Colloquium lecture; consulted with HAL on the Burgher's equation $u_t + uu_x = u_{xx} + u^p$.

3. Murray H. Protter, Mathematics Department, University of California, Berkeley. (4/18,19/85). Colloquium lecture; will consult with HAL about various ill posed prolems and eigenvalue problems.

4. Philip S. Crook, Mathematics Department, Vanderbilt University, Nashville (4/23/85). Colloquium lecture; will consult with HAL,TSC about numerical techniques.

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